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Article

Cultivating reasoning: Developing socio-mathematical norms for argumentation in form one algebra lessons in Mzilikazi District, Zimbabwe

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Abstract

The teaching of Form One Mathematics, particularly algebra, is largely rooted in algorithmic procedures. This formalic approach often hinders understanding and makes learning algebra difficult. Researchers note that The Heritage Based Curriculum, with its emphasis on the development of critical thinking and reasoning in Mathematics learning, does not explicitly recommend the adaptation of argumentation as a teaching approach. We addressed the research gap by implementing a pedagogical approach that sets out social norms for promoting argumentation in algebra lessons. The study adopted action research with six purposively selected teachers and used stratified sampling to select 180 learners according to their abilities. Data was gathered using audio recordings of 24 algebra lessons, learners' argument journals, pre- and post-tasks. The quality of the arguments were evaluated using Toulmin's argumentation model. Further, thematic data analysis revealed four learner justified socio mathematical norms: 'Convince your group', 'Mistakes are data', 'Use evidence', and 'Respectful disagreement'. Generalization with evidence increased from 10% to 45%. Pedagogical practices contributed to a 35% increase in learner particiaption and revealed the following changes; improved questioning strategies, using norm-referenced feedback, and using multilingual scaffolding. The study proposes a novel intersection between cultural philosophy and educational practice. Researchers conclude that the explicit norm-setting in lessons is an effective approach to turn algebra lessons into collaborative reasoning communities. The contextual application of argumentation in Form One Mathematics lessons enhances the understanding of algebra. Future research should focus on longitudinal studies that explore how socio-mathematical norms evolve, persist, or shift across different educational contexts.

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Introduction

Mathematics teaching in Zimbabwe faces several challenges, particularly in rural and under-resourced districts (Sunzuma & Luneta, 2022). The challenges include but are not limited to large class sizes, lack of teacher training in innovative pedagogical approaches, and limited access to teaching resources (Munyati et al., 2024). Additionally, traditional exam-oriented teaching methods often prioritize procedural approaches over conceptual understanding and reasoning skills (Sunzuma & Luneta, 2022; Lim et al., 2012). Mathematics teaching that emphasizes teacher-centred pedagogical methods implies that teachers and learners pursue correct answers. Schools tailor teaching and learning for success in examinations at the expense of understanding. The teachers' emphasis on the correctness of algebraic solutions and teacher-based approaches is likely to repress the development of learners' reasoning capabilities. The examination demands that teachers complete the syllabus does not foster sound critical thinking. Hence, the need to shift from teacher-centered teaching methods to learner-centered approaches, which encourage logical thinking and critical thinking are necessary, but remain elusive.

However, these are not the only challenges that inhibit effective teaching of algebra in schools. The district schools have enrolments that range between 50-60 learners. Faced with such enrolments, teachers fail to manage the large classes and to give individualized feedback. Linguistic diversity adds additional difficulty to the teaching and learning of mathematics. In the backdrop of multilingual contexts, learners cope through codeswitching between English, Shona and Ndebele when explaining themselves mathematically.

The Heritage-Based Curriculum (HBC) values reasoning, critical thinking, and communication. Authors believe that these are the major tenets of mathematics teaching and learning as prescribed by HBC. While so, at Form One level (entry grade for Secondary education), the learning of algebra is dominantly procedural, formulaic and is based on memorization. Adopting teaching and learning dependent on formulae and the emphasis on correct answers, prevents learners from getting to grips with algebraic concepts. Authors believe that it is imperative for the Zimbabwean Schools to address these issues to improve the quality of Mathematics teaching and improve learner academic performances.

We assert that the use of argumentation and socio-mathematical norms can effectively improve the teaching of mathematics. Nurruzzahra and Maarif (2024) define argumentation as the articulation of claims and provision of evidence. In support of these claims, Cobb and Yackel (1996) posit that norms are expectations about what constitutes a valid mathematical explanation. According to Cobb and

Yackel (1996) norms express the level of rigor required, and the criteria for evaluating mathematical solutions. Argumentation and socio-mathematical norms foster classroom environments in which learners feel encouraged to express their mathematical reasoning, justify their solutions, and constructively critique others' arguments.

Literature review

Conceptual framework

The study is informed by three theoretical frameworks: Toulmin's Argument Pattern (TAP), Krummheuer (1995)'s collective argumentation and Cobb socio-mathematical norms. These three frameworks offer meaningful ways to cultivate reasoning and argumentation in algebraic tasks, and they help shape the research questions around how socio-mathematical norms take root in mathematics classrooms. To begin with, Toulmin's Argument Pattern (TAP) (Toulmin, 1958) serves as a practical tool for both analysing and constructing arguments in educational settings. It breaks down an argument into parts that include claims, evidence, warrants, and backing, giving teachers a structured way to support learners in expressing their mathematical thinking more clearly. When learners engage with TAP, they become more confident in explaining their reasoning, justifying their solutions, and linking their claims to relevant evidence. In this way, TAP encourages thoughtful dialogue and critical engagement, both of which are vital for developing sound mathematical reasoning.

While Toulmin's model focuses on the internal structure of an argument, Krummheuer's framework shifts focus to the social dynamics that shape how arguments unfold in classroom interactions. It underscores the value of learners' participation in collective reasoning and critically evaluating the arguments of others. Krummheuer (1995)'s model supports the study's focus on socio-mathematical norms and demonstrates how collaborative discussions and agentic perspectives among learners can enhance an in-depth understanding of mathematical ideas. Krummheuer (1995) suggests that social approach promotes collaborative learning through the mediation of the facilitated collective argumentation. Therefore, teachers should play a facilitator role when conducting mathematical discourse and making learners explain their reasoning and justification of their proposed solutions.

Whereas Toulmin's work defines the argument structure, and Krummheuer's emphasizes collaboration, Yackel and Cobb (1996)'s work focuses on socio mathematical norms. According to Yackel and Cobb (1996), group norms develop

based on common consensus of what counts as valid mathematical reasoning in the classroom. The group norms support learners in developing a disposition toward the provision of evidence-based argumentation. In this approach, errors are valued and respect in argumentation is paramount. Cobb et al. (2001) asserts that socio-mathematical norms relate to mathematical activities, not to mathematical content. They are concerned with the evaluation criteria used on mathematical discourses. The framework sets the general targets for the development of socio mathematical norms aimed at improving argumentation and reasoning in the context of a supportive classroom environment.

Argumentation in mathematics education

Recent studies (Nurruzzahra & Maarif, 2024; Munyati, et al., 2024; Dimo, et al., 2021) identify argumentation as a requisite approach for mathematical reasoning. Argumentation fully explores mathematical concepts through constructing, presenting, and evaluating mathematical arguments to justify solutions. Researchers, such as Alexander (2017) and Munyati et al., (2024), have shown that argumentation strategies significantly improve learners' academic achievement and critical thinking skills. Cross-national research, e.g., Cobb & Yackel (1996) demonstrates that argumentation supports mathematics learning and deepening of thoughts, especially in algebra. Furthermore, Kieran (2007) revealed that involving learners in argumentation develops critical thinking and enhances understanding of mathematics. One of the important instruments for developing critical thinking is Toulmin (1958)'s model. By analyzing claims, warrants and rebuttals Toulmin (1958)'s model helps learners in building reasoned arguments. Despite the promise of argumentation, Munyati et al. (2024) posit that implementation of argumentation in mathematics classrooms with culturally diverse learners has often been resisted by those learners who are not comfortable with public criticism. Munyati et al. (2024) assert that this is characteristic of most African contexts and go on to suggest that in such contexts, careful facilitation is required. We argue that to create learning environments where learners feel comfortable engaging in argumentation without fear of stigma, teachers must recognise and value the cultural dimensions of their learners' lived experiences.

Socio-mathematical norms

A growing body of research highlights the power of dialogue and argumentation strategies in shaping socio-mathematical norms and strengthening learners' reasoning skills. Asterhan et al. (2018) and van Eemeren (2017) both point to the value of structured classroom discourse, where learners share their mathematical

ideas, justify their thinking, and engage critically with one another's arguments. In Zimbabwe, studies have shown that this approach can lead to significant improvements in learners' mathematical achievement (Munyati et al., 2024; Chitera et al., 2025).

While argumentation focuses on how learners construct and critique ideas, socio-mathematical norms, according to Cobb et al. (2001), go a step further by shaping what a classroom community considers to be a valid mathematical argument. These norms foster a culture of inquiry, where evidence-based reasoning is valued more than simply arriving at the correct answer. They encourage learners to embrace problem-solving, view mistakes as opportunities for growth, and participate in respectful critique (Cobb & Yackel, 1996). When teachers intentionally cultivate such norms, they help create learning environments where learners feel empowered to explain their thinking and engage in meaningful and collaborative discussions.

However, putting these ideas into practice isn't without challenges. One of which is the lack of teacher preparation. Many mathematics teachers in Zimbabwe according to Munyati et al. (2024) have not received sufficient training in facilitating classroom dialogue or nurturing argumentation skills. This gap underscores the urgent need for professional development programs that equip teachers with innovative strategies for fostering reasoning and discussion-based mathematics instruction. We add that class sizes and diverse learner abilities pose significant challenges in creating conducive environments for argumentation. Teachers often struggle to manage discussions for large classes and to ensure that all learners actively participate. Addressing these challenges requires targeted interventions, such as class size reductions and providing additional support for struggling learners (Munyati et al., 2024). According to Cobb and Yackel (1996) cultural norms and attitudinal tendencies of learners towards mathematics can also hinder the development of socio-mathematical norms. Chitera et al. (2025) assert that in some cases, learners may be reluctant to challenge their peers' arguments or to express their ideas due to fear of criticism or failure. Teachers must therefore create a supportive classroom culture that encourages risk-taking and which values diverse perspectives.

The teaching of algebra in schools

There is evidence in studies to date that suggest that teacher-controlled talk dominates mathematics classroom and is estimated at 78% (Makondo & Makondo, 2020). We point out that this approach hinders the learners'

engagement during lessons and hence limits the development of the argumentation practice. Despite the explicit focus on 'communicating solutions' in the Heritage Based Curriculum in Zimbabwe (MoPSE, 2024), a discrepancy between curriculum expectations and current classroom practices remains (Sunzuma & Luneta, 2022). Bethell (2016) and White-Fredette (2010) assert that teachers give routine tasks that learners are meant to solve through memorized procedures, and deductive reasoning. Each task usually has a single, static correct answer. In algebra, textbook content, classroom tasks and homework assignments are often decontextualized, inadequately seeking to develop learner's critical thinking. As an example, exercises such as 'Solve 5x + 2 = 17' or 'x-5=10' have no prompts for justifying or reasoning on the required solutions as suggested by Bersch (2019). The following figure shows such questioning drawn from the most used textbook at Form one level in Mzilikazi district.

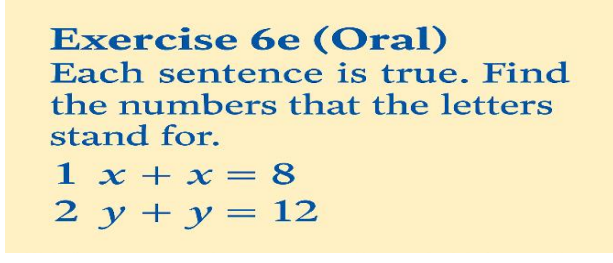

Figure 1. Example question from Form One textbook

This exercise falls short of the essential prompts that require learners to justify or give reasons to their solutions. Instead of asking learners to find out if the sentences are true, this exercise pre-empts that each sentence is true. Hence, it does not nature critical thinking so that through reasoning learners discover the truthiness of the sentences. This shortcoming is a barrier to the emergence of socio mathematical norms, and in turn deprives learners of participating in meaningful mathematical discourse. We believe that to address the curriculum-practice mismatch, teachers need to develop skills to teach learners logical thinking, and the use of interactive methods and critical thinking-oriented algebra teaching and learning in Zimbabwe.

Research problem

Several systemic challenges exist and inhibit the implementation of argumentation as a pedagogical strategy in Mzilikazi district secondary schools. The ten district secondary schools have at least 50 learners in each class. Such high enrolments limit teachers in having one-on-one interactions with learners, a necessity for a successful development of argumentation. The classes are diverse

mix of English, Ndebele Shona and other language speaking learners. Also, not all teachers are fluent in the different languages. The contexts complicate learners' linguistic landscapes as they must learn to mathematically argue in multiple languages. Furthermore, traditional teaching approaches predominantly used such as lecturing do not fit with the interactive and critical thinking approaches suitable for argumentation. Alexander (2017) asserts that many teachers lack experience in nurturing evidence-based discussions. Hence, they often revert to more familiar and teacher-centered 'demonstrate-imitate' learning cycles, which hinder deeper reasoning. Learners in the end do not do well in examinations and fail to pursue STEM-related careers, in which deep understanding of mathematics is a requirement.

Research questions

To address the challenges identified, this study responds to the following key research question:

- How can teachers establish socio-mathematical norms to cultivate argumentation in Form one algebra lessons?
- Three sub-questions that will help respond to the above main question are:
- What are the socio-mathematical norms that teachers/learners prioritize for argumentation in algebra lessons?
- How do teacher moves reinforce the socio-mathematical norms in algebra lessons?
- How do the socio-mathematical norms influence argument quality and participation in algebra lessons?

Methodology

Participatory action research

The study followed a Participatory Action Research (PAR) model as suggested by Kemmis et al. (2014). Researchers used the model to support teachers to implement an intervention design aimed at modifying their teaching practices. The PAR model fosters teacher collaboration and helps teachers to develop reflective practices. Action research stimulates the continual improvement of pedagogical strategies. The population of Form one mathematics teachers was 24 while that of learners was 465. Researchers purposively selected six Form One Mathematics teachers from three schools to participate in this study. The researchers used stratified random sampling to select 180 learners from the three secondary schools according to their academic performance.

The intervention design

The six teachers used argumentation strategies in a sequence of structured steps. Firstly, the collaborating teachers met to develop common teaching and research goals. Their discussions centered both on incorporating Toulmin's Argument Pattern and Cobb's socio-mathematical norms into their practice of teaching algebra. The adopted activities included structured group discussions, peer feedback, and exercises aimed at enhancing evidence-based thinking.

The researchers used an intervention design based on a Professional Development Cycle (PDC). This cycle reiterated teachers' knowledge about argumentation and socio mathematical norms through collaborative workshops and then practical work. The PDC addressed three essential stages of the intervention that included defining norms, co-designing activities, and creating the learners' tools. Figure 1 below shows the PDC sessions as implemented in the PDC.

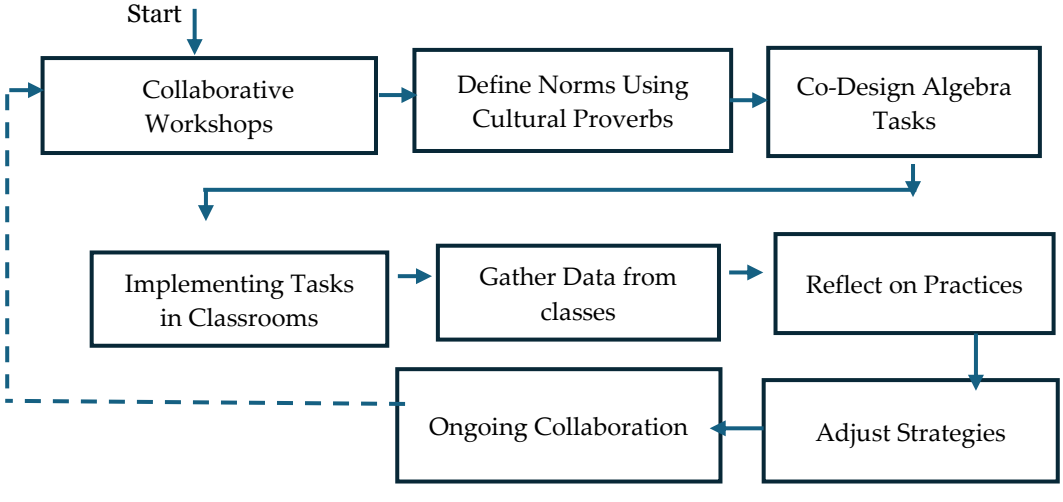

Figure 2. The PDC sessions

In the first session of the Professional Development Cycle (PDC), participants crafted norms using local (cultural) proverbs. The workshops assisted teachers to negotiate and describe classroom norms through local cultural knowledge in the guise of proverbs. Participants employed such proverbs to establish connections between cultural values and educational activities. For example, the adage 'Indlela ibuzwa kwabaphambili' (ask those who have travelled the path) when deliberated highlighted the need to seek support and evidence from the knowledgeable. The proverb was translated into the norm "Use evidence" in

mathematical reasoning. To help teachers create a culture in which learners felt safe to engage in the pursuit of evidence for their mathematical learning, participants learnt to use culture-based proverbs. Table 1 below shows the different proverbs that teachers deliberated on and used during the intervention.

Table 1. Socio cultural Ndebele proverbs

	Ndebele	**English translation**	**Interpretation**
1	Umuntu ngumuntu ngabantu.	Man is a product of human socialization	Encourages respect for others' ideas and **collaboration**
2	Izandla ziyagezana.	Hands wash each other.	Highlights the importance of **teamwork**. Foster a norm of helping one another in solving algebraic problems
3	Uhlanga lwendoda aluphumi emlonyeni.	One's success does not come from the word of mouth.	Emphasizes the importance of action over words. Encourage learners to back up their arguments **with actions and concrete examples** in their reasoning.
4	Isongo lilinye alikhali	A single bracelet does not jingle	Illustrates the value of **collaboration**

In the second phase of the PDC, the six teachers collectively created algebra tasks for developing argumentation and using reasoning and justification. One illustrative task that teachers designed was

Verify if $3(x - 4)$ is the same as $3x - 12$.

To encourage learners to think and reason, the task further asked learners to provide proof to their peers using words, numbers, and properties. Learners investigated equal expressions and had to talk through, i.e. explain themselves. Teachers participated in team-teaching, working together to design tasks aligned with the curricula but also encouraged critical thinking and argumentation. Teachers, using content co-creation approach, nurtured a sense of ownership and developed good classroom practices.

To promote learner engagement and guide argumentation, teachers who participated in the PDC, learnt to use additional tools in teaching algebra. These tools consisted of 'Convince Me!' Cards. These cards enabled the learners to receive peer feedback through prompts in local languages as well as in English. The cards supported learners to give feedback constructively and prompted them to express their reasoning and to engage in dialog with their peers.

In addition to the 'Convince me' cards, the teachers practiced using multilingual sentence starters to help learners communicate. Examples of sentence starters are:

"Ngicabanga ukuthi…", "Ndinofunga kuti", (I believe…)

"Ungangitshengisa njani?", "Ungandiratidze sei?" (How can you show me?)

These sentence starters supported learners in framing arguments and questions to make classroom discourse more accessible and inclusive. By providing linguistic scaffolding, it was the intention of the teachers to engage all learners, regardless of language ability, to help them engage in discussion. The inclusive PD cycle effectively provided teachers with culturally relevant norms, collaborative task design structures, and tools; as such, it intensified learner engagement and argumentation in algebra. This approach enhanced both the teaching methodology and collaborative engagements amongst learners. It created close connections among the learners and developed the learners' self-esteem in discussing their individual mathematical opinions.

Data collection and analysis

Across eight weeks of the intervention implementation, a variety of data sources were employed to holistically represent the landscape of teaching, learner argumentation, and the development of mathematical norms. To build a rich understanding of the research questions, we drew on a combination of classroom observations, audio recordings, samples of learners' work, and teacher reflections. Together, these methods provided a balanced mix of qualitative and quantitative data, allowing us to explore both the nuances of classroom interactions and broader patterns in teaching and learning of algebra.

In evaluating the impact of the algebra teaching intervention, we analysed records from 24 algebra lessons. These lessons offered insight into how teachers encouraged argumentation and supported the development of socio-mathematical norms through their instructional choices and classroom facilitation. We used lesson plans to find out what approaches and pedagogical goals teachers commonly used. Additionally, teachers kept reflective journals, providing immediate reflections in their own words regarding their thoughts, challenges, adaptations, and growth of their practices, and learner reactions, and their changing the way instruction was carried out.

The study investigated engagement in argumentation among learners through the administration of pre- and post-intervention written tasks. For example, to assess learners' knowledge of algebraic skills and ability to reason, learners were tasked to:

Explain why $2x + 2 = 2(x + 1)$.

The study examined the learners' responses for coherence and quality of reasoning and use of mathematical evidence. In addition, we also collected transcripts of whole class discussions and focused on learners' interactions from their negotiation of mathematical ideas and argumentation, such as challenging and supporting their peers' reasoning.

To monitor changes in socio-mathematical norms, learners responded to journals such as 'What is a good argument?' Prompted by these journals, learners both addressed their own understanding of argumentation, as well as the important attributes of effective mathematical reasoning. Learners also jointly produced 'norm negotiation posters', which visually represented consensual criteria for good arguments used in lessons. This routine promoted expression of understanding of argumentation, which in turn motivated ownership of the norms of the learner community.

The combination of methodological approaches generated a rich and multifaceted dataset that captured the realities of teaching and learning algebra in Mzilikazi district. This allowed for a close examination of how learners engaged with the material and how specific instructional strategies played out in practice. Ultimately, the findings contributed to a deeper understanding of how argumentation and reasoning skills can be effectively implemented, echoing insights from Bragg et al. (2018).

Regular teacher meetings played a vital role in this process. These collaborative sessions created spaces for teachers to reflect on the data, share experiences, and navigate both the challenges and successes of the intervention implementation. This ongoing dialogue supported real-time adjustments to teaching strategies and helped foster a community of practice grounded in mutual learning and professional growth.

To analyse the data, we employed two complementary techniques: discourse analysis and an argument quality rubric. Together, these tools provided insight into both the structure and substance of classroom interactions, allowing us to assess how learners constructed and communicated mathematical arguments. The two approaches facilitated the anlysis of both the quality of learners' arguments and the presence of socio normative discussions in classroom discourse.

Audio-recorded barrages and group discussion transcripts were analyzed using discourse analysis. Specifically, we used this process to systematically identify and

code example(s) of Toulmin's argument elements (claims, warrants, backing) in learner arguments. To reveal a socio mathematical norm count we coded behaviors and language that indicated a set of accepted norms for solving the tasks, including justification based on evidence, polite challenge, and treating mistakes as subversive. Using quantitative data, we measured frequency of use of the norms and their successful integration into the lesson discourse and identified areas to be improved.

To assess the quality of learners' mathematical arguments, the study employed a four-point rubric that captured the depth of evidence and reasoning demonstrated in each task. Each level of the rubric was carefully defined to reflect varying degrees of argumentative sophistication, as outlined in Table 2.

Table 2. Argument quality rubric

Level	Description	Example
1	States answer only	'Yes, they are equal'
2	Uses example/visual	'If x=1, both sides = 6'
3	Generalizes with evidence	'2x+4=2(x+2) by distributive property'
4	Anticipates counter arguments	'Holds unless x is undefined'

The rubric used offered a structured way to evaluate the quality of learners' mathematical arguments. At Level 1, responses lacked some form of explanation, offering minimal insight into the learner's reasoning. Level 2 responses included supporting detail and examples, indicating a greater level of engagement and an emerging ability to justify claims. At Level 3, learners demonstrated a more generalised understanding, using mathematical critique and showing familiarity with underlying principles. Level 4 responses went further, anticipating possible counter arguments and reflecting deep, thoughtful analysis and a strong grasp of the topic.

We applied this rubric to systematically code learner responses, allowing us to assess how the teaching intervention influenced the development of argumentation skills. This approach also helped us identify patterns in how learners' reasoning evolved over time. By combining this rubric-based analysis with other data sources, the study produced a detailed account of how

argumentation and socio-mathematical norms shaped algebra instruction in the participating classes and offered valuable insights for refining future teaching practices.

Findings

Over the course of eight weeks, teachers and researchers engaged in a collaborative exploration of how mathematical norms and argumentation practices unfolded within algebra classrooms. This joint effort allowed for reflective dialogue, classroom experimentation, and shared analysis—allowing both practitioners and the researchers to trace the evolving nature of reasoning and justification in learners' mathematical thinking. The following table shows the participant characteristics:

Table 3. Biography of participants

Role	Description
Teachers	4 Females, 2 Males; 5–20 years' experience
Learners	180 Form one learners. (Culturally diverse and stratified by performance)

The six participating teachers brought with them a diverse range of professional experiences, which enriched the study with varied pedagogical perspectives on how different instructional practices shaped the development of socio-mathematical norms and argumentation in algebra lessons. Their insights offered a textured understanding of classroom dynamics and the conditions under which mathematical reasoning flourishes. The 180 learners, purposefully stratified by performance levels, reflected a broad spectrum of cognitive profiles and socio-educational backgrounds. This diversity enabled a nuanced analysis of how argumentation and socio-mathematical norms influenced learners across different stages of algebraic understanding and engagement. Taken together, the composition of the participant group provided a robust and generative context for evaluating the impact of collaborative action research as a vehicle for deepening mathematical reasoning and fostering argumentation in secondary algebra classrooms.

Research question 1

This study identified four norms that teachers institutionalized in teaching algebra: peer persuasion, mistakes as data, evidence-based argument, and respectful disagreement.

Peer persuasion

In post-tasks, 78% of the learners practiced peer persuasion reflecting the 'Convince your group's norm. As an example, one learner expressed,

> I think we should use the distributive property in this case because it simplifies the equation and makes it easier to solve.

The learner cited above made an effort to convince the group by stating what they should do 'use the distributive property'. Furthermore, the learner attempts to justify use of the distributive property, that is first simplifies the equation, and that a simple equation is easy to solve.

Mistakes as data

In response to the prompt question: What information do we get from this mistake? one learner responded:

> When I made a mistake in calculation, I noticed that I forgot to distribute the negative sign. This taught me to double check my work.

We interpreted the above excerpt to mean that teachers' prompts helped learners to move toward the perspective of 'Mistakes are data'. Learners were learning from their mistakes as evidenced by performance that rose to 90% in post-intervention tasks.

Evidence-based argument

To develop evidence-based argument learners had to justify their solutions. One learner responded,

> I believe the solution is correct because when I substituted the value back into the original equation both sides were equal, proving my point.

Explicit evidence in arguments went up from 25% (pre-intervention) to 68% (post-intervention), supporting the norm "Use evidence".

Respectful disagreement

Following the introduction of the sentence starters, 92% of the learners' arguments included the use of "I" language (e.g., "I disagree because…"), indicating tolerance of dissent.

As an example, when teachers asked learners to explain differences in solutions, one learner explained,

> I differ from what my colleague said because if we look at the graph, it shows that the slope is actually negative, which supports my solution.

Across the eight weeks, learners learnt the shared criteria for what counts as a 'mathematically acceptable explanation', 'efficient strategy', or 'convincing justification'. These norms were co-constructed through classroom dialogue and teacher facilitation.

To cite one example, Learner C initially said, "x = 3 because I followed the steps", but revised the explanation after peers asked for justification: "I checked by substituting into the original equation, and it works". This is an extract from the observation Log, in Week 5.

Growth in algebraic argumentation

Learners' work and classroom talk showed an increase in the use of mathematical argumentation. Learners developed the ability to explain, justify, and critique solution approaches. Table 4 below shows how learners progressed in their reasoning.

Table 4. Argumentation development

Week	Argumentative move	Example
2	Claim without justification	"x = 4 because I divided by 2."
5	Claim + justification	"x = 4 because 2x = 8, and dividing both sides by 2 gives x = 4
8	Claim + justification + critique of alternatives	"x = 4 is better than factoring here because it's faster and avoids errors

The table shows that by week 2, the learners were still giving straight answers without reasoning. However, by week 5, their responses had justifications. By week 8, learners could critique alternatives. These shifts reflect learners' growing ability to engage in mathematical discourse and reason about algebraic procedures.

Research question 2

In response to the second research question number 2, teachers' pedagogical practices revealed the following changes; improved questioning strategies, using norm-referenced feedback, using multilingual scaffolding, and reflective practice.

Teachers showed improved questioning strategies. Asking straight forward questions such as 'What is the answer?' dropped by 70% while inquiry-based

questions such as 'How do you know?' increased by 85%, leading to more profound cognition.

Norm-referenced feedback was evident from all teacher participants. 80% of the teachers' feedback was explicitly based on norms as evidenced by increase in praises given to learners e.g., 'Good use of evidence, Ayanda!), reinforcing classroom expectations.

Additionally, during a lesson on solving equations, one teacher remarked,

I appreciate how you explained your reasoning, Thandi! Can you share how you arrived at that conclusion using our class norms?

The teacher cited above did not only acknowledge Thandi's effort but also encouraged her to articulate her thought process guided by the classroom expectations.

From the discussions between teachers and learners, there was evidence of multilingual scaffolding. Teachers skillfully employed codeswitching (e.g., Ndebele/Shona to English) to scaffold learners' reasoning. For instance, one teacher said, *Zvakakosha kuti udzidze izvi (It's important to learn this…) because it helps you to understand the problem better.*

Teacher's use of multiple languages enabled codeswitching and blending of the languages ensured that all learners grasped the concept. Reflecting on norms and discourse, teachers asserted that their learners began to seek explanations from their peers and challenging vague or incomplete reasoning presented. This shift in classroom culture was supported by the teacher's explicit modeling and feedback. One teacher reflected, "In Week 6, learners were inquisitive, asking each other, 'Why did you do that step?' or 'Can you prove it works?' without my prompting."

Research question 3

Table 5 below shows the results of the argument quality analysis.

Table 5. Argument quality

Level	Pre	Post
1	65%	20%
2	25%	35%
3	10%	45%

The researchers analysed the learners' responses in both the pre- and post-intervention tasks using the argument quality rubric. See Table 2. We contrasted

the rubric scores to assess how the instructional intervention affected argumentation skills and helped to track trends in learners' reasoning over time. The results showed that learners in level 1 in the pre-test were 65% but reduced to 20% in the post-intervention tasks meaning that learners moved from merely stating answers to justify their solutions. Twenty-five percent (25%) of the learners were in level 2 in the pre-test. In the post-test learners in level 2 increased to 35% meaning that there was an increase in learners giving examples or supporting evidence to explain their ideas. Ten percent (10%) learners were at level 3 in the pre-test tasks. The percentage rose to 45% in post-intervention tasks showing that other than generalizations, more learners provided explanations with mathematical justifications demonstrating understanding the algebraic concepts.

This study yielded a rich picture as to how teachers' use of argumentation and socio-mathematical norms when integrated in algebra lessons increase learners' understanding of concepts. Researchers believe that the multi-layered analytic approach used in the study offers opportunities for teachers to develop transformative and innovative teaching practices.

Discussion

The findings from the professional development cycle reveal significant transformative shifts in the teaching of algebra and classroom dynamics. This discussion examines these shifts in relation to existing literature, highlighting similarities, differences, and emerging themes.

Ubuntu philosophy aligned with collective reasoning

The integration of Ubuntu philosophy underscores the importance of community and interconnectedness, aligning well with existing literature on collaborative learning. Teachers and learners who adopt the principle that 'Mistakes are data' reduce the stigma associated with errors, echoing Vygotsky's emphasis on the social context of learning. By reframing mistakes as opportunities for growth, teachers create a learning environment that calls for openness and collaboration. According to Dweck (2006) positive classroom culture enhances learner engagement. However, while previous studies have acknowledged the value of a growth in mindset, the studies have not explicitly documented the application of Ubuntu philosophy in a multicultural context. Our study findings suggest a novel intersection between cultural philosophy and educational practice, prompting further exploration of how indigenous philosophies can inform modern pedagogical approaches.

Importance of multilingual scaffolding

The use of multilingual scaffolding as a means to democratize classroom participation aligns with literature advocating for inclusive teaching practices (García & Wei, 2014). Providing linguistic support through sentence starters and feedback prompts in multiple languages not only enhances comprehension but also fosters a sense of belonging among culturally diverse learners. This is consistent with findings of Baker (2011) that emphasize the importance of linguistic inclusivity in promoting equity. While literature highlights the benefits of multilingual education, this study illustrates a practical application in a specific context that may serve as a model for similar settings. The findings suggest that multilingual scaffolding not only aids in understanding but also strengthens community ties, further supporting the notion that education must be responsive to the cultural backgrounds of learners.

Effects of peer feedback tools

The introduction of peer feedback tools, such as the 'Convince Me!' cards, represents a practical innovation in collaborative learning. The findings indicate that these tools facilitate structured feedback processes, that enhance engagement even in large classroom settings. Study findings herein reported align with Hattie and Timperley's (2007) research on the power of feedback in learning, which emphasizes that effective feedback can significantly improve learner outcomes. However, the specific implementation of peer feedback tools in large classes presents a unique contribution to literature. While many studies examine feedback mechanisms, this research highlights the operationalization of peer feedback in a way that maximizes learner interaction. The positive effects observed suggest that such tools such the 'Convince Me!' cards can be essential in scaling collaborative learning, particularly in under-resourced environments where teacher-learner ratios may hinder individualized attention.

The findings underscore the transformative potential of embedding socio-mathematical norms and argumentation within algebra instruction. As learners engaged in collaborative problem-solving, they began to co-construct shared standards for what counts as valid mathematical reasoning. The shift marked a move away from mere procedural correctness toward valuing clarity, justification, and strategic thinking. Such evolution resonates strongly with sociocultural theory, which foregrounds the role of discourse and collective meaning-making in learning. The classroom became a space where mathematical knowledge was negotiated. It became a place where learners, through dialogue, refined their understanding of what constitutes a convincing argument or a sound solution.

The emphasis on socio-mathematical norms also reflected key constructivist principles: learners actively built understanding through argumentation, reflection, and peer interaction. Many transitioned from simply 'doing mathematics' to articulating and explaining their reasoning. This signaled deeper conceptual engagement and the emergence of mathematical identity. Yet, this transformation was not immediate. Some learners initially resisted the expectation to justify their solutions, opting instead for quick, unelaborated answers. In response, teachers scaffolded the process of argumentation, modelling how to critique ideas constructively and support peers in refining their reasoning. These pedagogical moves helped cultivate a classroom culture where explanation and critique became central to mathematical practice.

Emerging themes

The findings point to several compelling themes that emerged across the study: the importance of culturally responsive pedagogy, the central role of learning communities, and the impact of structured feedback in fostering meaningful learner collaboration. These insights align closely with contemporary educational thinking, which calls for holistic approaches that value learners' diverse backgrounds and lived experiences. In addition, the transformative shifts observed in algebra instruction, driven by the professional development cycle, highlight the potential of innovative teaching practices that reshape classroom dynamics in beneficial ways. As teachers continue to experiment with and refine these approaches, the implications for teacher preparation and curriculum design grow increasingly urgent and relevant.

Implications

The transformative shifts highlighted in this study carry weighty implications for educational policy, classroom practice, and future research. At the heart of these insights is a call to reimagine teacher training programs. Teachers' courses should embed dedicated modules on norm setting and mathematical argumentation. By equipping pre-service teachers with the tools to cultivate and sustain socio-mathematical norms, teacher education institutions can better prepare them to create learning contexts that emphasise reasoning, collaboration, and epistemic responsibility.

Practical strategies such as the *Convince Me!* cards, when used as daily exit tickets, offer a simple yet powerful way to reinforce peer feedback and reflective thinking. This routine encourages learners not only to articulate their own reasoning but to engage critically with the arguments of their peers. Likewise,

starting lessons with norm reminders such as, 'Today, we value evidence!', especially when framed through culturally resonant proverbs, can help embed a classroom culture of argumentation which is both intellectually rigorous and culturally grounded.

Such consistent reinforcement is key to making evidence-based reasoning a natural part of learners' mathematical experiences. To deepen these insights, future research should focus on longitudinal studies that explore how socio-mathematical norms evolve, persist, or shift across different educational contexts. Understanding the long-term sustainability of these norms will offer valuable guidance for embedding argumentation and reasoning in mathematics education in ways that are both culturally responsive and pedagogically sound.

Limitations

While the study revealed encouraging outcomes, its scope was shaped by a relatively small group of participating teachers and learners. The limitation may affect how broadly the findings can be applied across varied educational contexts. Expanding the sample size in future research could offer deeper, more informed insights into the effectiveness of the interventions and their adaptability in diverse settings. It's also important to acknowledge the potential influence of observer presence during audio recordings. Knowing they were being observed learners and teachers may have subtly changed how they interacted, possibly leading to classroom behaviours that differ from typical day-to-day dynamics. Despite these constraints, the study depicts meaningful shifts in practice and opens up promising directions for enhancing argumentation and reasoning in algebra classrooms. At the same time, it highlights the need for continued inquiry and refinement particularly in developing strategies that are both contextually grounded and sustainable over time.

Conclusion

Developing socio-mathematical norms for argumentation in Form One algebra classrooms across at secondary school, calls for a layered and intentional approach, one that blends teacher training with collaborative teaching and learning practices. While challenges such as large class sizes and cultural barriers remain, the study shows that targeted interventions and innovative pedagogies can help navigate these complexities and foster classroom cultures rooted in reasoning, argumentation, and critical thinking.

The findings reveal that when socio-mathematical norms are co-constructed and culturally adapted, they hold transformative potential for algebra instruction in

the secondary schools. Grounding these norms in local cultural contexts and actively involving teachers in collaborative professional development demonstrates how meaningful shifts in classroom dynamics can emerge from within.

When teachers deliberately nurture mathematical reasoning, they create contexts where learners move beyond rote memorization and begin to engage as active, evidence-driven thinkers. The shift is not only pedagogically significant but equips learners with the analytical tools needed for deeper mathematical understanding. It prepares them to tackle real-world problems with confidence. The emphasis on argumentation and reasoning also strengthens learners' critical thinking and collaborative skills, which are essential across academic and everyday contexts.

As Zimbabwe continues to implement its Heritage-Based Curriculum (HBC), scaling these approaches will require strategic investment in teacher agency and the development of multilingual resources. Empowering teachers to take ownership of their instructional choices enhances their responsiveness to innovation and to the diverse needs of learners. At the same time, ensuring access to multilingual materials supports inclusive participation in mathematical discourse, allowing all learners to engage meaningfully regardless of linguistic background. Ultimately, the study underscores the potential of socio-mathematical norms and argumentation, not only in enriching algebra teaching and learning in Zimbabwean classrooms, but also in offering adaptable models for other culturally diverse educational settings.

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Appendices

Appendix A: Sample Argumentation Task

This appendix presents a sample argumentation task, which prompts learners:

a) Is $5x - 10$ equivalent to $5(x - 2)$?
b) Solve $3x + 5 = 20$. For both justify using:
   (i) A test value
   (ii) A property of operations
   (iii) A written explanation designed to convince a skeptic"

Appendix B: Learner Journal Template (Ndebele/English)

This appendix provides the template for the learner journal, presented in both Ndebele and English.

The Ndebele version includes the following prompts:

- Igama lami: _____ (Name: _____)
- Namuhla ngifunde: _____ (Today I learned: _____)
- Ngichaze kanjani ukuthi impendulo yami iyiqiniso? _____ (How I justified my answer: _____)

Appendix C: Norm Negotiation Protocol

This appendix outlines the protocol followed for norm negotiation. The steps are as follows:

1. Brainstorm answers to the question: "What helps us learn mathematics deeply?"
2. Cluster the brainstormed ideas into cohesive norms.
3. Vote to select the top four norms.
4. Create posters illustrating each selected norm with clear examples and non-examples.

Appendix D: 'Convince Me!' card used in a classroom setting

Name: _____
Date: _____
Task:
a) Is 5x - 10 equivalent to 5(x - 2)? Justify your answer.

1. Claim:
Write your answer (Yes or No):

2. Reasoning:
Explain why you think this is true or false:

3. Evidence:
Provide an example or calculation to support your claim:

4. Counterargument:
How would you respond to someone who disagrees with you?

Remember:

- Use respectful language.
- Support your arguments with evidence.
- Listen to your peers' viewpoints.